Exponential, power and log functions

- Prior learning required
  - Squares + cubes
  - Square + cube roots
  - Exponents
  - Distributive property
- Review laws of exponents ➔
  - Power function, $y = ax^n$
    - $y = 6x^2$
  - $a^m \times a^n = a^{m+n}$
    - $10^3 \times 10^2 = 10^{3+2} = 10^5$
  - $a^m \div a^n = a^{m-n}$
    - $10^5 \div 10^3 = 10^{5-3} = 10^2$
  - $a^0 = 1$
  - $(a^m)^n = a^{mn}$
  - $1/a^n = a^{-n}$
  - Rational $a^{m/n} = \sqrt[n]{(a^m)}$
    - for $a > 0$
- Building inverses ➔
  - f(x), inverse is $f^{-1}(x)$
  - Exchange x, y co-ordinates
  - Exchange x, y variables
  - Function and its inverse, reflect across y = x
- Keywords
  - Function
  - Exponent
  - Base
  - Power
  - Root
  - Logarithm
    - Common logarithm = base 10
      - $\log_{10} x = \text{Log } x$
    - Natural logarithm = base e
      - $\log_e x = \ln x$
- John Napier (1550 – 1617) Philosopher, inventor of the decimal point, inventor of napier's bones for mechanically doing multiplications, divisions and finding roots... and inventor of logarithms.

- Log functions
  - Exponential form
    - $4^3 = 64$
    - $y = b^x$
  - Logarithmic form
    - $\log_4 64 = 3$
    - $\log_b y = x$
- Inverse of $y = b^x$
  - $f(x) = b^x$
  - $f^{-1}(x) = \log_b y$
- Properties of log functions
  - Product property
    - $\log xy = \log x + \log y$
      - $\log 12 = \log(3 \cdot 4)$
      - $= \log 3 + \log 4$
      - $= \log 3 + \log 2^2$
      - $= \log 3 + \log 2 + \log 2$
      - $= \log 3 + 2 \log 2$
  - Quotient property
    - $\log {}^x/_y$
      - $= \log x - \log y$
  - Power property
    - $\log x^n = n \log x$
    - $\log x^3 = \log(x \cdot x \cdot x)$
      - $= \log x + \log x + \log x$
      - $= 3 \log x$
- Change of base
  - To change a log in base b to a log in base a
    - $\log_a x$
      - $= \log_b x \div \log_b a$
      - Derivation:
        - Express x in terms of two bases a and b
        - Let $x = a^m$, and let $x = b^n$
        - So, $m = \log_a x$ and $n = \log_b x$
        - $x = a^m = a^{\log_a x}$
        - Subsititute for x in $\log_b x$
        - $\log_b x = \log_b(a^{\log_a x})$
        - $\log_b x = \log_a x \log_b a$
        - $\log_a x = \log_b x \div \log_b a$
    - To change from base 10 to any other base a, let b = 10
      - $\log_a x$
        - $= \log x \div \log a$
      - To find $\log_2 100$
        - $\log_2 100$
        - $= \log_{10} 100 \div \log_{10} 2$
        - $= 2 \div 0.3010299957$
        - $= 6.64385619$
        - $2^{6.64385619} = 100$
- e
  - The base of the natural logarithm, $\log_e$ or ln
  - e = 2.178281828...
- Another way to think about this...
  - $\log_{10} 2 = 0.301029996$
  - Or, $10^{0.301029996} = 2$
  - Note: $0.301029996 = {}^{301029996}/_{100000000}$
  - So, $10^{.301029996} = \sqrt[100000000]{10^{301029996}}$
  - Or, $\log_{10} 2$ is the hundred-millionth root of ten raised to the power of 301,029,996!